PROM/SE Promoting Rigorous Outcomes in Mathematics and Science Education

Research Report Vol. 1, 2006 Michigan State University

Making the Grade: Fractions in Your Schools

What do you see when you look in math classrooms in the schools in your district? State guidelines in place. Approved textbooks. Good lesson plans. Well-trained, hard-working teachers. Eager, hard-studying students doing their best. Their report cards carry reasonable grades home to their parents. Yet, many of those students are failing while we watch without seeing.

What more can we do? You get the best teachers, the best books, meet tough standards and have hard-working students. They are doing their best and we are doing our best for them. Or are they? Are we? No. In many important ways, they are not doing their best and neither are we. They are not learning the mathematics they could learn, should learn. They are not getting ready for a world peopled by other students who are learning mathematics elsewhere even now. They are not learning enough to prepare them for the world they will face. They are not getting a chance to do all that they are capable of. In important ways, they are not making the grade even while they make their grades.

Our students must do better in mathematics. How can we make this happen? How can we help them really make the grade? We can't do everything; we must do the important things. What is important? What needs fixing? What can we do about it?

Understanding fractions is one important area where we are not doing all we can do. It is one of the most important topics in all of elementary and middle school mathematics. It is important for its own sake. It is also important for other parts of mathematics. For example, learning algebra is much harder without understanding fractions well.

Understanding fractions is also difficult. It is one of the most difficult topics for students to learn. Why is it so hard for many students to understand fractions at a satisfactory level? The PROM/SE data offer some important clues. They help make it clear why understanding fractions is so hard. They also suggest some solutions that may help overcome the problem. Among the important things they reveal, more of which will be given later, are:

- **Third grade is likely the problem: little important learning in crucial areas of fractions takes place there.**
- **Large numbers of students are not learning foundations like equivalent fractions and common denominators.**
- **Little more about fractions will be learned in high school yet students are being sent on from eighth grade without adequate knowledge.**

The PROM/SE test data were collected two years ago. They were collected from all of the students in your district. About 200,000 students in 60 districts across Ohio and Michigan were tested. This included students from third through twelfth grades. Your district may not have included all of those grades but all of your students were tested. We look here at some of what we learned from that testing.

 Supported by the National Science Foundation under cooperative agreement EHR-0314866

What has PROM/SE learned about what students can do?

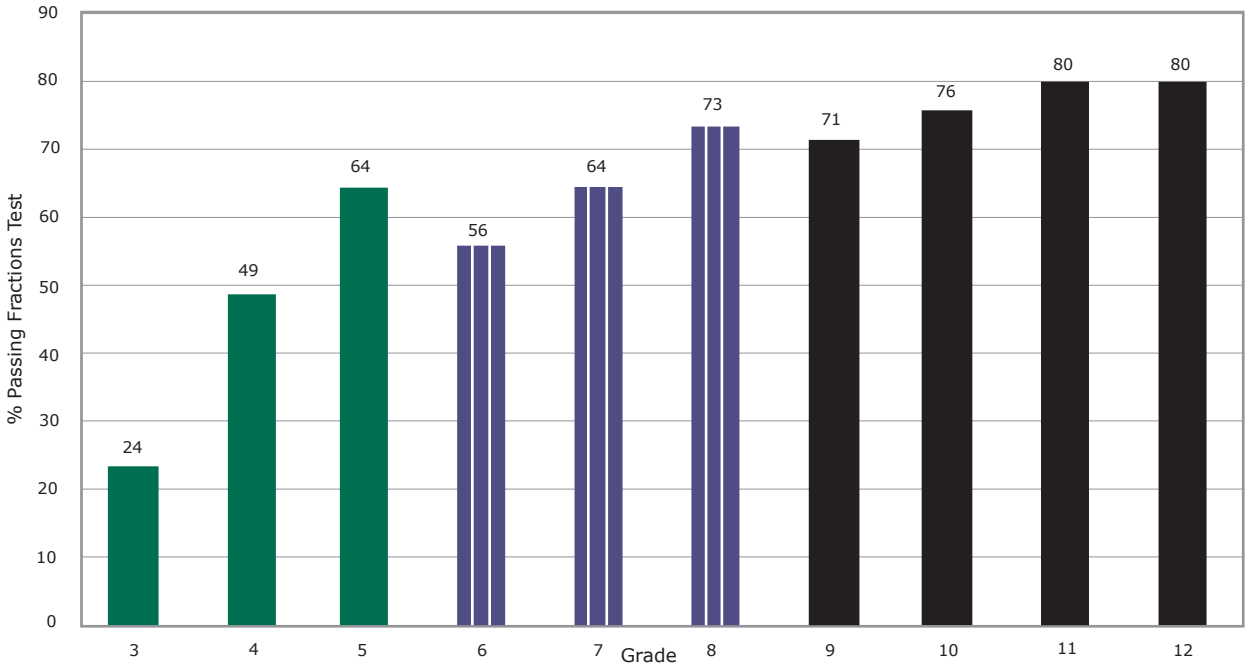

Figure 1. Percent Passing Fractions Test by Grade

Look at Figure 1 above. It shows the "learning curve" across third through twelfth grades for the fractions test[1]. This is for all students in the 60 districts. It shows the percentage of students who make a passing mark on the fractions test at each grade. What does this tell us?

Elementary School

• The percentage of students who pass the test steadily increases in third through fifth grades.

• About an additional 20 percent pass the test in each of fourth and fifth grades.

• The percentage passing the test drops from fifth grade to sixth grade. Most likely, this reflects that the form of the test changed starting with sixth grade.

• About 35 percent of Michigan and Ohio PROM/SE students still cannot pass the elementary fractions test by the end of fifth grade.

• About 60 percent of third graders in the top-achieving countries passed the TIMSS fractions test, a test comparable to the PROM/SE test. However, only about 25 percent of the PROM/SE third graders passed their test.

Middle School

• The percentage passing continued to increase in sixth through eighth grade, but only about 10 percent per grade — not as strong an increase as that for fourth and fifth grades.

• Some big gains are still made in the middle school years. About 20 percent more can pass the test in eighth grade than could pass it in sixth grade.

• Even by the end of eighth grade, still about 25 percent cannot pass the fractions test.

High School

• The passing percentage increased in high school but only by a small amount. The combined gain over four years was only about an additional 10 percent.

[1] A common test was used for third through fifth grades. A different common test was used for sixth through eighth grades. Finally, a third common test was used for ninth through twelfth grades. Passing is defined in terms of the domain (fractions) using the international results from TIMSS (the Third International Mathematics and Science Study, 1995) as benchmarks. Failure is defined as answering less than 50 percent of the items that currently define the domain of fractions.

 Supported by the National Science Foundation under cooperative agreement EHR-0314866

What does this mean?

• The greatest gain in the number of students passing the fractions test takes place in third through sixth grades. The TIMSS 1995 results suggest this is true worldwide. So, it should not be a surprise that this is true in Michigan and Ohio. That is the good news.

• The bad news is how low the passing rate is in the elementary grades! This seems to come from the low passing percentage starting in third grade. Growth in fourth and fifth grades is reasonably good, so the problem is the starting point.

• Every district ought to ask itself: "Is our third grade fractions curriculum challenging enough? Is the work in earlier grades adequate?" Are children gaining strong conceptual frameworks? The answers seem to be "no" compared to worldwide results.

• The mathematics curriculum moves to the set of rational numbers after sixth grade. Understanding fractions by then is crucial for all students if they are to succeed. **But almost half of the students in these 60 districts are not passing the fractions test by this critical point!**

• Those who are not able to pass the test by the end of eighth grade are not likely to do so by the end of high school. In neither Michigan nor Ohio does the high school curriculum focus on fractions. Unsurprisingly there are few additional gains in the passing percentage during those years. This leaves around 20 percent of students without the understanding necessary to pass this test even when they graduate from high school. This is in addition to the other problems they will have with high school mathematics caused by not knowing this material when they enter high school. They will be left behind in a very basic mathematics topic, one that is important for everyday life.

• The critical point in the learning curve seems to be third through sixth grades. To increase what students learn in fractions, the focus should be the curriculum in those grades, especially third grade and even before. **How fractions are taught in the second and third grade is a critical starting point for an upward curve leading to satisfactory success in learning fractions.**

• Almost half of the students entering seventh grade cannot pass the middle grades fractions test. This has to have an impact on seventh and eighth grade mathematics. More time is currently needed for fractions in those grades because of what is not learned in earlier grades. Because that time has to be spent on fractions, seventh and eighth grade mathematics cannot move on to geometry and algebra as is done in most of the rest of the world.

PROM/SE Associates are mathematics and science teachers who participate in intensive professional development trainings offered by PROM/SE.

 Supported by the National Science Foundation under cooperative agreement EHR-0314866

What does this mean for my district?

	% Passing									
District	3	4	5	6	7	8	9	10	11	12
A	10.5	40.4	58.6	45.1	59.0	67.7	81.4	71.5	79.6	69.1
AA	21.7	39.9	66.1	36.1	47.0	49.8	56.3	62.5	74.8	69.7
AB	22.2	42.9	51.6	36.8	57.8	67.8	62.9	68.4	73.5	75.0
AC	22.6	71.8	64.8	55.3	63.2	75.0	78.6	74.2	77.9	72.0
AD				50.0	59.1	68.1				
AE	23.7	32.6	51.7	39.8	48.3	59.1	66.7	68.9	78.9	79.0
AF	24.4	42.7	74.1	55.4	71.8	76.9	74.3	75.8	88.0	84.2
AG	26.1	68.2	83.3	72.6			74.8	76.8	78.3	77.8
AH	27.5	74.3	84.5	78.1	83.3	86.7	77.7	88.8	91.5	91.9
AI	28.7	53.0	70.5	58.0	62.7	67.8	68.6	66.6	75.6	69.0
AJ	30.4	49.0	81.6	57.1	48.4	75.3	74.6	83.3	78.9	77.6
AK	30.9	68.5	79.0	67.1	74.2	82.0	79.1	80.8	80.3	84.6
AL	31.8	62.3	79.3	66.4	62.0	69.9	70.1	80.8	79.8	90.0
AM	32.8	68.4	79.4	70.6	78.1	89.2	87.3	87.2	91.1	95.5
AN	33.3	69.9	86.4	55.6	80.2	87.0	76.4	83.1	82.6	84.8
AP	38.6	49.4	80.0	50.3	57.8	62.9	70.6	79.4	78.6	90.6
AQ	39.8	81.2	90.6	81.9	83.8	85.9	81.3	85.3	90.8	84.5
AR				51.9	66.3	79.9				
AS	45.6	92.4	93.4	72.9	84.0	82.4	87.8	84.0	87.6	89.9
AT	48.5	75.3	85.4	66.7	76.3	79.6	84.0	76.0	91.1	92.0
AU	50.0	75.9	80.9	72.4	74.8	88.8	84.5	88.3	92.3	86.8
AV	53.9	82.0	91.7	69.5	84.3	83.0	82.1	85.6	90.3	85.7
AW				69.6	86.8	92.0				
AY	4.4	18.2	43.1	57.7	50.0	56.3	46.0	62.1	85.4	77.6
AZ	7.0	34.2	64.9	30.1	40.7	58.3	59.7	56.8	67.3	68.1
B	11.5	35.6	52.5	34.8	46.5	51.8	54.6	64.2	72.3	72.9
BA	0.0	55.9	71.2	42.0	46.2	51.2	59.3	60.8	70.2	67.9
BB	8.8	44.4	73.7	53.1	66.7	82.2	67.1	74.6	80.4	75.7
BD	23.5	56.5	69.8	55.6	64.3	75.2	61.7	68.3	74.9	75.9
BE				47.8	62.7	85.6				
BF	41.6	69.4	87.2	66.3	72.4	77.6	72.6	74.4	82.7	85.3
BG	18.7	37.9	69.2	40.3	43.2	53.1	69.2	76.1	84.9	79.0
BH				48.0	71.0	81.0				
BI				61.5	74.6	78.6				
BJ	60.1	87.4	92.2	76.6	65.0	84.1	79.2	79.4	83.3	82.6
BK									73.4	79.7
C	14.4	39.5	68.3	52.8	52.4		61.8	68.7	68.6	65.2
D	15.1	35.9	48.2	46.0	52.5	60.3	60.7	67.3	72.7	64.7
E	20.2	40.6	66.5	57.8	72.4	86.1	83.8	83.6	80.2	85.8
F	22.1	40.2	49.2	53.3	55.8	59.2	57.7	63.5	66.1	69.5
G	25.0	52.9	68.5	49.5	67.9	68.9	70.3	62.7	63.9	85.2
H	28.3	59.4	83.8	48.3	61.7	69.2	62.8	70.2	66.9	79.4
I	28.9	52.7	70.0	59.1	59.4	74.6	74.3	75.8	77.5	80.7
J	29.6	69.3	77.2	61.5	70.0	79.7	79.2	71.3	77.5	76.9
K	29.8	63.4	80.5	65.0	73.2	84.9	79.9	83.4	84.0	87.1
L	36.8	68.1	84.8	57.9	74.6	81.3	73.7	79.9	84.2	86.5
M	39.0	73.8	82.3	64.9	77.2	86.4	80.7	83.7	85.5	84.1
N	51.8	78.7	88.8	61.7	81.7	81.2	78.6	72.2	84.0	79.2
O	57.9	78.0	86.5	71.0	71.8	86.0	81.3	87.9	88.7	85.5
P	9.8	29.2	41.4	45.2	43.1	42.6	47.7	52.6	54.9	57.8
Q	9.8	46.3	62.5	44.0	50.0	62.3	75.6	74.7	57.6	77.7
R	10.5	34.6	44.5							
S	12.7	16.8	27.7	34.7	51.3	50.0	46.3	56.5	58.5	65.1
T	13.8	33.7	47.0	35.3	54.4	65.9	67.2	76.8	78.4	67.8
U	16.1	45.1	56.5	48.5	64.5	71.2	73.8	86.7	85.2	80.6
V	16.7	43.1	62.9	39.9	56.9	83.3	72.2	86.0	85.9	86.1
W	16.7	58.9	64.8	55.9	55.3	81.1	75.0	81.2	81.0	70.2
X	17.3	28.0	78.8	46.9	53.6	69.6				
Y	17.8	82.2	98.0	79.6	70.3	71.1	76.8	79.0	85.1	82.6
Z				56.4	67.5	77.6				

Table 1. Percent Passing Fractions Test by District and Grade

• The results and comments in the previous section are general and apply across the PROM/SE partnership. They have similar implications for most districts. However, some districts have fraction learning curves that are very different from the overall curve.

• Here is an example:
If District Q were your district, only 9.8 percent of the third graders passed the test. In fourth grade, 46 percent passed the test. At fifth grade, 63 percent could pass the test. The learning rate is approximately 25 percent per year. In fact, even though third graders in this district started out lower than typical for the districts, because of a higher-than-average learning rate they essentially had the same typical percent passing by sixth grade. For them, clearly the important issue is what happens in third grade. What happens in their curriculum at that grade? How confident and knowledgeable are the third grade teachers about these topics? How effective is their instruction? Imagine what would have happened if a larger percentage of third-grade students had passed the test – say 25 percent (roughly the third grade average) — because of a coherent third grade curriculum that emphasizes student understanding and fundamental fraction meanings. Assuming the same rate of growth, by the end of fifth grade 75 percent would have passed the test!

 Supported by the National Science Foundation under cooperative agreement EHR-0314866

What learning takes place in third through fifth grades? With which areas of fractions do students have difficulty?

Because fractions are so important in elementary school mathematics, the PROM/SE elementary test provides more than just a total fractions test score. It also provides scores on seven distinct fractions subtests. These subtests measure the following sub-areas of fractions:

1. Models of fractions: part/whole, set, length and area
2. Equivalent fractions and mixed numbers
3. Ordering fractions (including the use of the number line)
4. Adding and subtracting fractions (most items with the same denominators with just a couple of exceptions where the two different denominators involved are 2 and 4)
5. Multiplying fractions
6. Word problems involving fractions
7. Common denominators (including least common denominators and adding fractions with unlike denominators)

Table 2 presents the percentages of students in the PROM/SE districts passing the seven subtests for third through fifth grades.

	Grade		
Subtest Area	**3**	**4**	**5**
Models	61	81	88
Equivalent Fractions	23	46	60
Ordering Fractions	25	49	62
Adding and Subtracting	24	46	60
Multiplying	10	23	35
Word Problems	26	48	62
Common Denominators	5	13	23

Table 2. Percent Passing Subtest Areas

• The percent passing in third grade in four of the subtests in Table 2 is very similar to the overall test as seen for third grade in Figure 1. However, students do better on the models subtest (61 percent passing). They do much worse than the overall on the multiplying and the common denominator subtests.

• Lower passing percentages at third grade on the multiplying fractions and the common denominator subtests are not surprising. These topics typically are not covered at the third grade, even internationally. However, by the end of fifth grade only 35 percent and 23 percent pass these subtests. That is both surprising and something for concern.

• The common denominator subtest also has very low passing rates at fourth grade. This further suggests how difficult this area is. This particularly low passing rate is somewhat unexpected because working with common denominators is related conceptually to equivalent fractions for which students did better, with almost half passing at fourth grade. This raises questions of coherence – the degree to which the logical structure of the mathematics linking various fraction topics influences instructional activities and is made visible to students.

• The fifth-grade passing rate for the common denominator subtest is extremely low. This is serious. This content is basic in other areas such as adding and subtracting fractions with unlike denominators and ordering fractions. It is also essential for working with proportions in the middle grades. These data say serious change is needed in this area at fifth grade where it is called for in both the Michigan and Ohio standards. By the end of fifth grade only about one fourth of the students can pass this subtest.

• Sixty percent of the fifth graders can pass the subtest for adding and subtracting fractions. How does this relate to the low passing percent for the common denominators subtest? The adding and subtracting subtest uses fractions with "like" denominators at these grades (except the simple case of unlike denominators 2 and 4). So, the adding and subtracting subtest does not involve finding difficult common denominators.

• Sixty percent of the items on the sixth grade overall fraction test do involve finding common denominators. Students do not do well on this test. This likely is because they have not mastered common denominators by this grade (as Table 2 shows). At fifth grade, only 23 percent of the students pass the subtest on common denominators. Together these results are a flashing red warning light.

 Supported by the National Science Foundation under cooperative agreement EHR-0314866

What does all this mean?

• The biggest problem with students learning fractions is the elementary school curriculum, not the middle school curriculum. Often the middle school curriculum is blamed but that is most likely because this is where the problem with fraction understanding especially shows itself. Further learning in mathematics is limited by the middle grades for a large number of students because of their problems with fractions.

• It would be ideal if all students (or at least 90 percent) could pass the fractions test by the beginning of seventh grade. This is where the curriculum moves on to more advanced topics. Many of these topics depend on an understanding of fractions. The reality is only about half of the students pass.

• There are at least two approaches to reaching the 90 percent goal — either raise the initial third grade passing rate or increase the learning rate over fourth, fifth and sixth grades. Both seem desirable. Table 1 shows that both can be done, because at least some districts demonstrate that one or both approaches work. Raising the initial third grade passing rate stands out most clearly.

• State standards call for ordering fractions and equivalent fractions at third grade. However, too few students pass subtests in these areas at this grade. It may be that students are not getting an adequate opportunity to learn these topics. That is, there may be a problem with the fractions curriculum in third grade. It does not seem to provide a solid foundation for growth in fourth and fifth grades.

• Introducing models of fractions is going on and going well. Over 60 percent of third-grade students passed that subtest. The problem is the other two areas.

• Remember that, on a comparable test, about 60 percent of the students in the top-achieving countries around the world passed the test at third grade. If your district could achieve this passing rate at the third grade then, assuming the rate of gain over the following grades, we can project that by the end of fifth grade essentially all students would pass the elementary fractions test.

• Common denominators is the other big weakness in covering fractions, especially at fifth grade. It is in the state standards. Students do reasonably well in the equivalent fractions subtest, which is related to this content but that does not show how serious the problem is. There are extremely low passing rates for the common denominators subtest in third and fourth grades. Clearly that is a warning of how soon the problem starts that will be so serious in fifth grade. It seems very likely that this problem is related to weakness with whole number multiplication tables.

• The percent of students who fail fractions overall at eighth grade remains relatively constant from then on. It is about 20 to 25 percent throughout high school. A large number of students are left behind. High school mathematics courses do not seem to help this.

What should I do to help?

• Locate your district data in Table 1 and study your own district's results carefully. Discuss the data with district curriculum experts, mathematics teachers and PROM/SE Associates.

• Little important learning in crucial areas of fractions takes place in third grade according to the test data. Examine your textbooks and instructional materials. Do they address the fractions sub-areas adequately, especially in third grade? State standards call for covering those sub-areas but test data show we are not getting the job done. Find out from teachers what they are doing with fractions in third grade.

• Foundations like equivalent fractions and common denominators are not being successfully learned by large numbers of students. Focus on stronger work on ordering fractions, understanding common denominators and finding equivalent fractions in the early grades. This may relate to lack of fluency with basic multiplication facts.

• Work with models of fractions is being successful in second grades and third grades. Include more work with models of equivalent fractions in those grades to represent the idea of common denominators.

• Raise expectations for student performance in adding and subtracting fractions more demanding beginning in third grade and push for conceptual depth in this area.

 Supported by the National Science Foundation under cooperative agreement EHR-0314866

• Fourth grade is probably one of the most important grades in the arc of mastering fractions. Getting more done here can change things in later grades. More demanding work with adding and subtracting fractions, even those with more complicated denominators, should help in the long run. Work with models for common denominators, simple ones in second and third grades and more complicated ones in fourth and fifth grades.

• Put more attention on multiplying fractions in fifth grade and especially in sixth grade. Relate multiplying to fraction models (see Figure 2) if it is not already being done in the earlier grades.

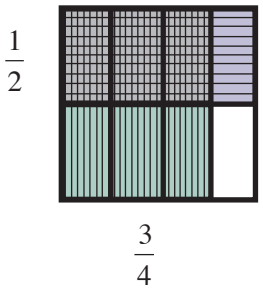

Figure 2. Area Model for $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

• Intensify work with word problems in third and fourth grades. Relate the word problems to fraction models if that is not already done.

• Little more about fractions will be learned in high school, yet students are being sent on from eighth grade without adequate knowledge. Develop a strategy for eighth grade and earlier so more eighth grade students can pass a fractions test. If possible, spread this strategy over several grades. Make it a strategy, not a specific class, so that these students are exposed to more demanding content in other areas and not lost in a remedial class ghetto. Fractions are important for other areas of mathematics. For example, lack of understanding of fractions is a particular problem for eighth grade algebra because ideas of line and slope are central in beginning algebra. Do not send students on to high school without conquering this content.

Conclusion

These are just some of the important ideas highlighted by the PROM/SE data for fractions and some of their implications. These data need to be the focus of thought and discussion in your district. Improving your students' ability to work with fractions is important to improving their overall mathematical skills. Solutions are not simple but are important. They must be more than band-aids if we are to end some of the silent hemorrhages in our mathematics classrooms.

PROM/SE, Promoting Rigorous Outcomes in Mathematics and Science Education, provides cutting-edge professional development to K-12 teachers and administrators in participating Michigan and Ohio schools. Educators meet throughout the year as part of the one-day mathematics workshop series, two-day science institutes and the intensive summer academies to delve into specific topics in mathematics and science.

Visit **www.promse.msu.edu** for a full calendar of PROM/SE learning opportunities.

PROM/SE by the Numbers:
• 60 school districts
• about 1000 Associate Teachers
• 5000 in-service teachers
• 800 pre-service teachers
• 300,000 K-12 students
• over 500 contact hours of professional development delivered to date

PROMSE
Promoting Rigorous Outcomes
in Mathematics and Science Education

 Supported by the National Science Foundation under cooperative agreement EHR-0314866

MICHIGAN STATE UNIVERSITY

Promoting Rigorous Outcomes in Mathematics and Science Education (PROM/SE) is a comprehensive research and development effort to improve mathematics and science teaching and learning in grades K-12, based on assessment of students and teachers, improvement of standards and frameworks, and capacity building with teachers and administrators.

PROM/SE Partners:
Calhoun ISD, MI
High AIMS Consortium,OH
Ingham ISD, MI
Michigan State University
SMART Consortium, OH
St. Clair County RESA, MI

Co-Principal Investigators:
Joan Ferrini-Mundy
William Schmidt
George Leroi
Peter Bates
Terry Joyner

The PROM/SE RESEARCH REPORT is published by PROM/SE, Michigan State University, 236 Erickson Hall, Michigan State University, East Lansing, MI 48824.

Send inquiries to:
Susan Pettit Riley, editor
(517) 353-4884,
promse@msu.edu
www.promse.msu.edu

The PROM/SE RESEARCH REPORT is based on work supported by the National Science Foundation under agreement EHR-0314866. Any opinions, findings and conclusions or recommendations expressed in this report are those of the authors and do not necessarily reflect the views of the NSF.

What do state standards expect for fractions?

Ohio

• The Ohio Mathematics Indicators call for introducing commonly used fractions in first grade. These include halves, thirds and fourths. They are introduced using physical models. Second grade emphasizes fractions as parts of a whole. It also includes comparing and ordering physical models of halves, thirds, and fourths. These are ordered in relation to 0 and 1.

• Third grade focuses on representing fractions and mixed numbers using words, numerals and physical models. Comparing and ordering fractions is a focus using the number line, physical models and equivalent forms.

• Fourth grade introduces adding and subtracting commonly used fractions with "like" denominators. It also includes estimating computations involving fractions. Fractions are related to whole numbers and decimals.

• Fifth grade focuses on equivalent fractions and common denominators. It also focuses on adding and subtracting commonly used fractions with "like" and "unlike" denominators. Fractions are rounded (including mixed numbers) to the nearest half.

• Sixth grade focuses on meanings of fractions —the division of whole numbers, ratios and so on. It also focuses on multiplying and dividing fractions using models and visual representations. Another goal is fluently using algorithms for computing with fractions. The concept of rational number is introduced.

Michigan

• The Michigan Grade Level Content Expectations call for beginning work with fractions in the second grade. This is mostly at the conceptual level.

• That work continues into third grade. It includes using the number line. It includes whole units being partitioned into parts of equal area or length. Equivalent fractions are introduced. Mostly these have denominators of two, four or eight. Ordering fractions is also introduced. So is adding and subtracting fractions with "like" denominators.

• Fourth grade expands work with equivalent fractions to fractions with other denominators. Improper fractions and mixed numbers are introduced. More complicated adding and subtracting fractions is included. Multiplication of fractions by whole numbers using repeated addition and area models is introduced.

• In fifth grade, students multiply fractions using an area model. Division is related to repeated subtraction with whole numbers and to fractions. Adding and subtracting fractions with unlike denominators is introduced.

• Sixth grade further develops concepts related to dividing fractions. Division is presented as the inverse of multiplication. Multiplying or dividing any two fractions, including mixed numbers, fluently is a goal. Rational numbers are introduced.

Both States

• Both the Ohio and Michigan state standards call for introducing fractions in second and third grades. This includes defining fractions, equivalent fractions and ordering fractions. (Michigan also includes adding and subtracting fractions with like denominators.) These are three (four in Michigan) of the seven sub-areas used to make up the fractions test.